The schwarzschild radius

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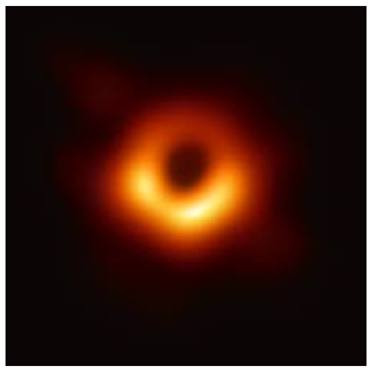
abstract:

Black holes come in different sizes, ranging from stellar black holes that are a few times the mass of our Sun to supermassive black holes that are millions or billions of times more massive than the Sun and can be found at the centers of most galaxies. While they cannot be directly observed, their presence can be inferred from their effects on nearby matter and the way they bend light around them. Black holes are fascinating objects that continue to captivate scientists and the public alike with their mysterious and extreme nature.

In this paper we will dive into the basics concepts regarding the physics of a black hole, calculating the swharzschild radius for M87* and give a two dimensional visualisation of a black hole and the accretion disk as well as simulate photons falling into a M87*.

1 Introduction

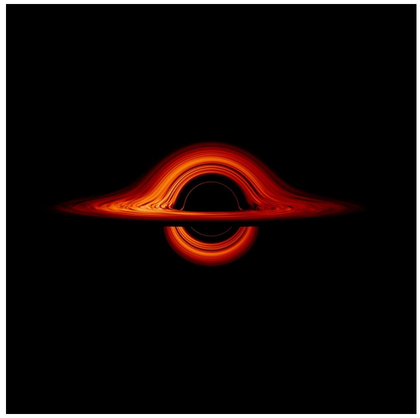
A black hole is a region in space where the gravitational pull is so strong that nothing, including light, can escape from it. It is formed when a massive star runs out of fuel and collapses under its own gravitational pull, compressing its mass into an infinitely small point called a singularity. This singularity is surrounded by a region called the event horizon, which marks the point of no return beyond which anything that enters will be consumed by the black hole's immense gravity.



On April 10, 2019, the Event Horizon Telescope collaboration published the first photo of a black hole, the one at the center of the M87 galaxy. This black hole has a mass of about 6.5x10^9 Ms,where Ms is the mass of the sun, which is approximately 2x10^{30} Kg. What we see in the picture above is hot matter outside the horizon of the black hole.

2 Accretion disk

An accretion disk is a disk-shaped region of gas, dust, and other debris that surrounds a central object, in particular, a black hole. The gas and other material in the disk are gradually pulled inward by the object's gravitational attraction, eventually forming a disk-like structure that spins around the central object.



The accretion disk is an important concept in astrophysics, as it plays a crucial role in a wide range of astronomical phenomena, including the formation of stars and planets, the accretion of material onto black holes and other compact objects, and the emission of light and other forms of radiation from these objects.

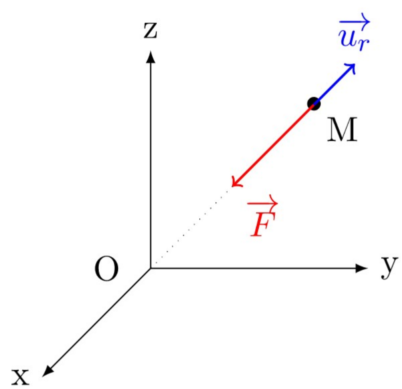
3 Scharzschild's radius

3.1 Escape velocity

The escape velocity is the minimum velocity that an object needs to escape the gravitational pull of a celestial body, such as a planet or a star, and move into space. In other words, it is the speed required to overcome the gravitational force of the celestial body and leave its surface without falling back down.

3.1.1 Centrifugal conservative forces

we will consider first a purely classical model in which the newtonian force is considered a centrifugal force.



In this paragraph we will express the Schwarzschild radius in terms of the black hole's mass. To simplify the problem, we will consider the model to be , later on we will add the relativistic corrections.

Let us consider an object with a mass M at the point O, M is

The mechanical energy is defined as : $E_m(r) = E_p(r) + E_k(r)$ Where E_p and E_k are respectively the potential and kinetic energies.

The gravitational potential energy is determined by integrating the newtonian force. $E_p(r) = \frac{-GMm}{r}$. Where as E_k is simply equal to $\frac{mv^2}{2}$

The escape velocity is determined such that $E_m(r) = 0$. Therfore $\frac{-GMm}{r} + \frac{mv^2}{2} = 0$

hence : $v^2 = \frac{2GM}{r}$.

Now, if we consider the object M to be a photon, its velocity would be equal to c the speed of light. We would have the following : *

$$\boxed{r_s = \frac{2GM}{c^2}}$$

4 A 2D visualisation of a black hole

4.1 P5JS : A powerful and easy to use javascript framework

There are many reasons why

It's easy to learn: p5.js was designed to be accessible to people who are new to coding. The syntax is straightforward and easy to understand, and there are lots of tutorials and examples available online to help you get started.

It's versatile: p5.js can be used to create all kinds of interactive projects, from simple animations to complex data visualizations. It's also compatible with a wide range of platforms and devices, so you can create projects that work on desktop computers, mobile devices, and even in virtual reality environments.

It's open source: p5.js is an open source project, which means that anyone can contribute to its development and improvement. This has led to a vibrant community of users who share code, offer support, and collaborate on projects.

It's fun: Perhaps the most important reason why p5.js is awesome is that it's a lot of fun to use! Whether you're creating interactive art, building games, or exploring data in new ways, p5.js offers endless possibilities for creativity and experimentation.

4.1.1 The code

please see the attached js file.